Name	Date
AP Calculus AB	

Summer Assignment 2023 - 2024

Order of Topics:

- A. Complex Fractions
- B. Simplifying Rational Expressions
- C. Solving Equations
- D. Solving Absolute Value Equations
- E. Piecewise Functions
- F. Area (Calculator Allowed)
- G. Volume & Surface Area (Calculator Allowed)
- H. Functions
- I. Even & Odd Functions
- J. Intercepts
- K. Points of Intersection
- L. Interval Notation
- M. Inequalities
- N. Domain & Range
- O. Graphs of Common Functions
- P. Logarithms
- Q. Transformations/ Graphing
- R. Exponents
- S. Factoring
- T. Completing the Square
- U. Solving Quadratic Equations
- V. Inverses
- W. Equation of a Line
- X. Radians & Degrees
- Y. Angles in Standard Position
- Z. Reference Triangles
- AA. The Unit Circle
- **BB.** Graphing Trigonometric Functions
- CC. Inverse Trigonometric Functions
- DD. Right Angle Trigonometry
- **EE.** Solving Trigonometric Equations
- FF. Circles
- GG. Polynomial Long Division/ Synthetic Division
- HH. Sum & Difference Formulas, Double-Angle Formulas, & Half-Angle Formulas
- II. Calculator Questions (Calculator Allowed)

A calculator should only be used on topics (above) OR individual problems in the packet that state "Calculator Allowed".

You are responsible for knowing and understanding all of the topics in this packet (they are all from Algebra I & II, and Pre-Calculus). I would suggest starting by completing every other or every few problems, and then going through and doing the rest of the problems for any of the topics you have trouble with. This packet itself will not be graded. In September, we will have two days, as a class, to go over any topics you are having trouble with (so 96 minutes of class time review) and then you will have a two-day test on this content that will count as your first test grade in marking period 1. I will send out the answer key with all the work included by the beginning of August so you can review it yourself and come in, ready with questions.

A. Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction. Simplify what remains.

For $\#s\ 1-10$, simplify each of the following (get rid of complex fractions). Write only positive exponents in your final answer.

$$1. \ \frac{\frac{25}{a} - a}{5 + a}$$

$$2. \quad \frac{2 - \frac{4}{x + 2}}{5 + \frac{10}{x + 2}}$$

$$3. \quad \frac{4 - \frac{12}{2x - 3}}{5 + \frac{15}{2x - 3}}$$

4.
$$\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$$

$$5. \quad \frac{1 - \frac{2x}{3x - 4}}{x + \frac{32}{3x - 4}}$$

6.
$$\frac{x^{-3}+x}{x^{-2}+1}$$

7.
$$\frac{\frac{x-5}{2x-5}}{\frac{x-5}{2x-5}\frac{x+2}{x-5}}$$

$$8. \quad \frac{\frac{y^2}{4}}{\frac{y}{y+4}}$$

9.
$$\frac{\frac{1}{2}(2x+5)^{-2/3}}{\frac{-2}{3}}$$

10.
$$\frac{(x-1)^{1/2} - \frac{x(x-1)^{-1/2}}{2}}{x-1}$$

B. Simplifying Rational Expressions

When adding or subtracting fractions, find a common denominator. Combine the numerators and keep the denominator. Reduce, if possible.

When multiplying fractions, factor both the numerators and denominators and cancel out any common factors. Multiply across and reduce, if possible.

When dividing fractions, keep-switch-flip. Then, follow the rules of multiplication.

For #s 11 - 17, simplify the expression.

11.
$$\frac{12}{x+2} - \frac{4}{x}$$

$$12.\,\frac{2x-1}{x-1}-\frac{3x}{2x+1}$$

$$13.\,\frac{15x+18}{3}\cdot\frac{x+1}{30x^2+36x}$$

$$14.\,\frac{2x^2-8x-64}{8x^2+32x}\cdot\frac{5x+6}{5x^2+x-6}$$

$$15. \frac{12x^2 + 8x}{2x^2 + 3x - 54} \div \frac{12x^2 + 8x}{2x^2 - 25x + 72}$$

$$16.\,\frac{27x+18}{9x^2}\div\frac{21x^2-7x-14}{14x-14}$$

$$17.\frac{2x-3}{49-x^2} \div \frac{2x^2+5x-12}{x^2-3x-28}$$

C. Solving Equations

When solving equations that involve fractions, find the least common denominator (LCD) and multiply every term by the LCD. When you do that, all the fractions disappear, leaving you with an equation that is hopefully solvable. Answers should be checked in the original equation for extraneous solutions. (You should check for extraneous solutions with rational equations, absolute value equations, and equations where you raise both sides to an even power.)

For #s 18 - 24, solve the equation. Check for extraneous solutions.

$$18. \frac{x}{2x-6} - \frac{3}{x^2 - 6x + 9} = \frac{x-2}{3x-9}$$

$$19. \, \frac{5}{2x} - \frac{5}{3(x+5)} = \frac{5}{x}$$

20.
$$\sqrt{x+2} = x - 4$$

$$21.\frac{2}{3}(x-1) + \frac{1}{4}x = 10$$

$$22.\ 2\sqrt{x} - \sqrt{2x+1} = 1$$

23.
$$2(x-5)^{-1} + \frac{1}{x} = 0$$

$$24. x^4 + x^2 - 6 = 0$$

D. Solving Absolute Value Equations

To solve an absolute value equation, isolate the absolute value expression. Set the quantity inside the absolute value notation equal to both the positive and negative quantity on the other side of the equation. Solve for the unknown in both equations. Check your answer(s) in the original equation.

For #s 25 - 27, solve the absolute value equation. Check for extraneous solutions.

25.
$$|x + 5| + 5 = 0$$

26.
$$|x - 10| = x^2 - 10x$$

27.
$$|x| + |2x - 2| = 8$$

E. Piecewise Functions

The definition of the absolute value function is a piecewise function: $f(x) = |x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$ where x is called the "argument".

To rewrite an absolute value equation as a piecewise function, find where the argument equals 0. Put those values, the critical numbers, on a number line and test numbers in each interval to see where the argument will be positive and where it will be negative. Write the results as a piecewise function.

For #s 28 - 31, rewrite the absolute value equations as piecewise functions.

28.
$$f(x) = |2x + 4|$$

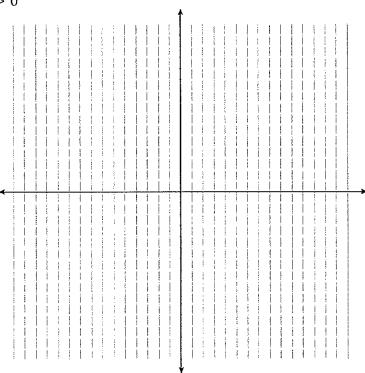
$$29. f(x) = |2x^2 + 5x - 3|$$

$$30. f(x) = |3x - 9| + 2$$

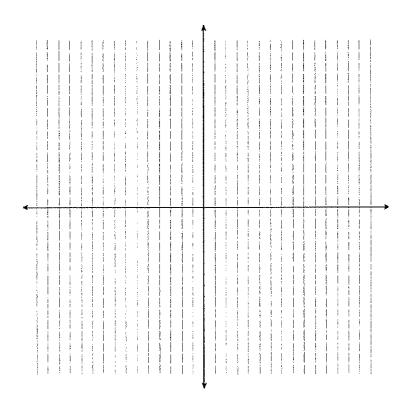
31.
$$f(x) = |(x+2)(x-4)|$$

For #s 32 - 34, graph the piecewise functions.

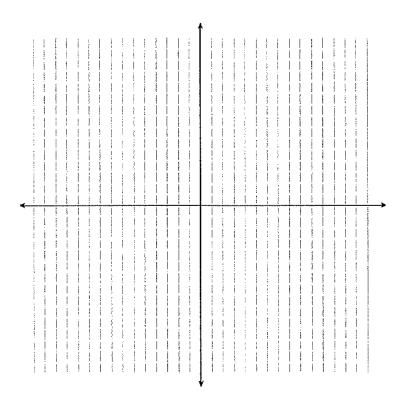
32.
$$f(x) = \begin{cases} x^2 + 4, x \le 0 \\ 3x - 5, x > 0 \end{cases}$$



33.
$$\begin{cases} x^2 - 4, x \le 1 \\ 5, 1 < x < 2 \\ -x, x \ge 2 \end{cases}$$



34.
$$\begin{cases} -x^2 + 8, x < 2\\ (x - 4)^2, x \ge 2 \end{cases}$$



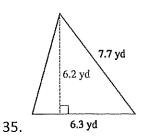
F. Area (Calculator Allowed)

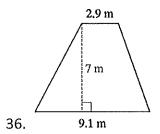
Area Formulas:

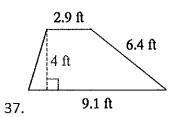
- Area of a Square: $A = s^2$ or A = lw or A = bh
- Area of a Rectangle: A = lw or A = bh
- Area of a Triangle: $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
- ullet Area of a Parallelogram: A=bh
- Area of a Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
- Area of a Circle: $A=\pi r^2$
- Circumference of a Circle: $C=\pi d$ or $C=2\pi r$

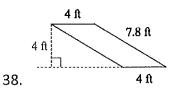
s=side, l=length, w=width, b=base, h=height, r=radius, d=diameter $b_1 \otimes b_2$ are the parallel bases in a trapezoid

For #s 35 - 38, find the area of each figure.

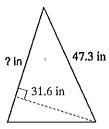




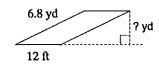


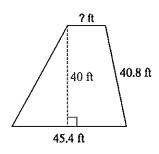


For #s 39 - 41, find the missing measurement given the area.



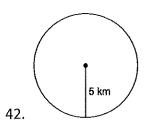
39. Area = 714.2 in^2

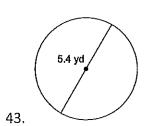




Area = 1208 ft^2

For $\#s\ 42-43$, find the circumference and area of the circle.





44. If the circumference of a circle is $59.1 \ km$, find the area of the circle.

45. If the area of a circle is $98.02\ cm^2$, find the circumference of the circle.

G. Volume & Surface Area (Calculator Allowed)

Volume Formulas:

• Volume of a Prism (or a Cylinder): V = BH

• Volume of a Pyramid (or a Cone): $V = \frac{1}{3}BH$

• Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

B = area of the base, H = height of the figure, & r = radius

Surface Area Formulas:

• For most figures, add the areas of all the surfaces.

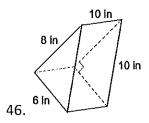
• Surface Area of a Cylinder: $SA=2\pi rh+2\pi r^2$

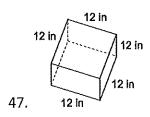
• Surface Area of a Cone: $SA=\pi r ig(r+\sqrt{h^2+r^2}ig)$ or $SA=\pi r m{l}+\pi r^2$

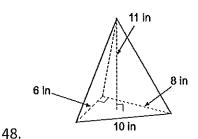
• Surface Area of a Sphere: $SA = 4\pi r^2$

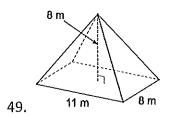
r = radius, h = height, l = slant height

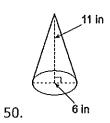
For #s 46 - 54, Find the volume of each figure.

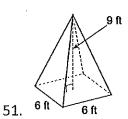


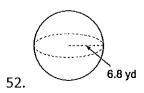


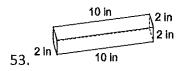


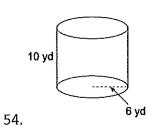




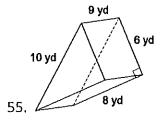


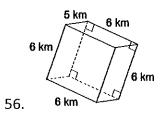


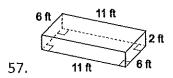


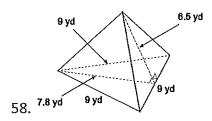


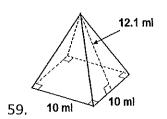
For #s 55 - 63, find the surface area of each figure.

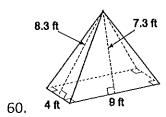


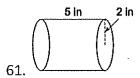


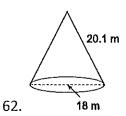












H. Functions

A function is a set of points (x, y) such that for every x, there is one and only one y.

To evaluate a function for a given value, plug the value into the function for x.

Recall that $(f \circ g)(x) = f(g(x)) = f[g(x)]$, read "f of g of x", means to plug the inside function in for x in the outside function.

64. Let
$$f(x) = 3x^2 - 5$$
 and $g(x) = \frac{2}{x}$. Find $f(g(x))$.

65. Given
$$f(x) = 2x - 1$$
 and $g(x) = x^2 - 1$, determine $g(f(x))$.

For #s 66 - 72, let f(x) = 2x + 1 and $g(x) = 2x^2 - 1$.

66. Find f(2).

67. Find g(-3).

68. Find f(t + 1).

69. Find $\left(\frac{g}{f}\right)(x)$.

70. Find f[g(-2)].

71. Find g[f(m+2)].

72. Find
$$\frac{f(x+h)-f(x)}{h}$$
.

For #s 73 – 74, let $f(x) = \sin x$. Use exact values.

73. Find
$$f\left(\frac{5\pi}{6}\right)$$
.

74. Find
$$f\left(\frac{4\pi}{3}\right)$$
.

For #s 75 - 78, let
$$f(x) = x^2$$
, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$.

75. Find
$$h[f(-2)]$$
.

76. Find
$$f[g(x-1)]$$
.

77. Find
$$(f + h)(x)$$
.

78. Find
$$g[h(x^3)]$$
.

For #s 79
$$-$$
 81, let $A(r) = \pi r^2$.

79. Find
$$A(2s)$$
.

80. Find
$$A(3)$$
.

81. Find
$$A(r+1) - A(r)$$
.

For #
$$s$$
 82 $-$ 83, find $\frac{f(x+h)-f(x)}{h}$ for the given function f .

82.
$$f(x) = 9x + 3$$

83.
$$f(x) = 5 - 2x$$

For #s 84 - 86, let
$$f(x) = \begin{cases} x^2 - 3, & x \ge 0 \\ 2x + 1, & x < 0 \end{cases}$$

84. Find f(5).

85. Find
$$f(2) - f(-1)$$
.

86. Find
$$f(f(1))$$
.

For #s 87 - 88, let
$$\begin{cases} \sqrt{x+2} - 2, & x \ge 2 \\ x^2 - 1, & 0 \le x < 2 \\ -x, & x < 0 \end{cases}$$

87. Find
$$f(0) - f(2)$$
.

88. Find
$$\sqrt{5 - f(-4)}$$

For #s
$$89 - 93$$
, let $f(x) = x^2 - 5x + 3$ and $g(x) = 1 - 2x$.

89. Find
$$(f \circ g)(x)$$
.

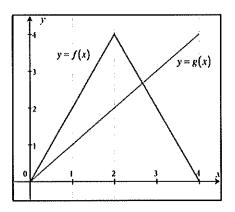
90. Find
$$(f - g)(3)$$

91. Find
$$f(g(3))$$
.

92. Find
$$f\left(\frac{2}{3}\right)$$
.

93. Find
$$(fg)(x)$$

For #s 94 - 96, use the graphs of f(x) and g(x), given below.



- 94. Find g(3).
- 95. Find x such that f(x) = 4.
- 96. Find x such that f(x) = 2.

I. Even & Odd Functions

Functions that are even have the characteristic that for all a, f(-a) = f(a). What this says is that plugging in a positive number, a, into the function or a negative number, -a, into the function makes no difference; it produces the same result. Even functions are symmetric to the y-axis.

Functions that are odd have the characteristic that for all a, f(-a) = -f(a). What this says is that plugging in a negative number – a into the function will give you the same result as plugging in the positive number and taking the negative of that. Odd functions are symmetric to the origin.

If a graph is symmetric to the x-axis, it is not a function because it fails the Vertical Line Test.

For #s 97 - 99, show that the following functions are even.

97.
$$f(x) = x^{2/3}$$

$$98. f(x) = \left| \frac{1}{x} \right|$$

99.
$$f(x) = x^4 - x^2 + 1$$

For #s 100 - 102, Show that the following functions are odd.

100.
$$f(x) = x^3 - x$$

$$101. f(x) = \sqrt[3]{x}$$

102.
$$f(x) = e^x - e^{-x}$$

103. Determine if
$$f(x) = x^3 - x^2 + x - 1$$
 is even, odd, or neither. Justify your answer.

J. Intercepts

To find any x-intercepts, set y = 0 and solve.

To find the *y*-intercept, set x = 0 and solve.

For #s 104 - 109, find the x- and y-intercepts for each.

104.
$$y = 2x - 5$$

105.
$$y = x^2 + x - 2$$

106.
$$y = x\sqrt{16 - x^2}$$

107.
$$y^2 = x^3 - 4x$$

$$108. f(x) = e^x$$

$$109. g(x) = \ln x$$

K. Points of Intersection

Use the substitution or elimination method to find the solution(s) to the equation, thus, finding the point(s) of intersection.

For #s 110 - 112, Find the point(s) of intersection on the graphs (or the solutions to the equations) for the given equations.

110.
$$x + y = 8$$

 $4x - y = 7$

111.
$$x^2 + y = 6$$

 $x + y = 4$

112.
$$x^2 - 4y^2 - 20x - 64y - 172 = 0$$
$$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$$

L. Interval Notation

Description	Interval Notation	Description	Interval Notation	Description	Interval Notation
x > a	(a,∞)	$x \le a$	$(-\infty, a]$	$a \le x < b$	[a, b)
$x \ge a$	[<i>a</i> ,∞)	a < x < b	(a, b)	$a < x \le b$	(a, b]
x < a	$(-\infty,a)$	$a \le x \le b$	[a, b]	All real numbers	(-∞,∞)

If a solution is in one interval or in another, interval notation will use the connector \cup .

For #s 113 - 115, complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
113.		
$-2 < x \le 4$		
		And the second s
444		
114.		
	[-1,7) ∪ [9,∞)	
	[1,7) 0 [2,00]	
115.		
		21/2
		-5 -4 -3 -2 -1 0 1 2 3 4 5
	1	
		ST.

M. Inequalities

Solving inequalities is the same as solving equations. If you multiply or divide by a negative number, you must switch the sign of the inequality.

To solve linear absolute value inequalities:

- 1. Isolate the absolute value.
- 2. Identify what the absolute value inequality is set "equal" to.
 - a. If the absolute value is less than zero, there is no solution.
 - b. If the absolute value is <u>less than or equal to zero</u>, there is <u>one solution</u>. Just set the argument equal to zero and solve.
 - c. If the absolute value is greater than or equal to zero, the solution is all real numbers.
 - d. If the absolute value <u>is greater than zero</u>, the solution is <u>all real numbers except for value</u> which makes it equal to zero. This will be written as a union.

- e. If the absolute value is <u>less than or equal to a negative number</u>, there is <u>no solution</u>. The absolute value of something will <u>never</u> be less than or equal to a negative number.
- f. If the absolute value is <u>greater than or greater than or equal to a negative number</u>, the solution is <u>all real numbers</u>. The absolute value of something will <u>always</u> be greater than a negative number.
- g. If the absolute value is <u>less than or less than or equal to a positive number</u>, set the argument less than the number and greater than the opposite of the number using an "and" statement in between the two inequalities. The solution will be written as an intersection.
- h. If the absolute value is greater than or greater than or equal to a positive number, set the argument less than the opposite of the number and greater than the number using an "or" statement in between the two inequalities. Then, solve each inequality, writing the solution as a union of the two solutions.

To solve rational inequalities:

- 1. Write the inequality as an equation.
- 2. Solve the equation.
- 3. Determine all values that make the denominator zero.
- 4. Draw a number line, and mark all the solutions and critical values from steps 2 and 3.
- 5. Select a test point in each interval between the marks on the number line, and determine if they satisfy the original inequality.
- 6. If a test point satisfies the inequality, then every point on that interval does. That interval should be identified in the solution.
- 7. State your answer including all intervals that solve the inequality.

For #s 116 - 124, solve each inequality. State your answer in both interval notation and graphically.

116.
$$2x - 1 \ge 0$$

117.
$$-4 \le 2x - 3 < 4$$

118.
$$\frac{x}{2} - \frac{x}{3} > 5$$

119.
$$|x - 15| \ge 5$$

120.
$$|x-2|-3 \le 2$$

$$121. \qquad \frac{x+5}{|x-2|} \le 0$$

122.
$$\frac{3x+2}{(x+1)(2x)} \le 0$$

$$123. \qquad \frac{x+1}{2} \ge \frac{1}{x}$$

124.
$$\sqrt{x+4} \ge \sqrt{x-2}$$

N. Domain & Range

The domain of a function is the set of allowable x-values. The domain of a function f is $(-\infty, \infty)$ except for values of x which create a zero in the denominator, an even root of a negative number, or a logarithm of a non-positive number.

The range of a function is the set of allowable *y*-values.

For #s 125 - 140, find the domain and range of each function. Write your answer in interval notation.

125.
$$f(x) = x^2 - 5$$

126.
$$f(x) = -\sqrt{x+3}$$

$$127. f(x) = 3\sin x$$

128.
$$f(x) = \frac{2}{x-1}$$

129.
$$y = \frac{x^2+4x+6}{\sqrt{2x+4}}$$
 (find domain only)

130.
$$y = \log(2x - 10)$$

131.
$$f(x) = \ln(4 - x)$$

132.
$$y = \frac{\sqrt{2x-9}}{2x+9}$$

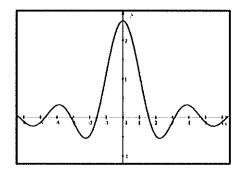
133.
$$y = \sqrt{x^2 - 5x - 14}$$

134.
$$f(x) = \frac{x^2 - 5x - 6}{x^2 - 3x - 18}$$

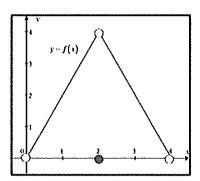
135.
$$f(x) = \frac{1}{x^2 - 1}$$

136.
$$f(x) = \sqrt{x^2 - 4}$$

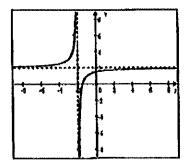
137.
$$f(x) = |x^2 - 4|$$



138.

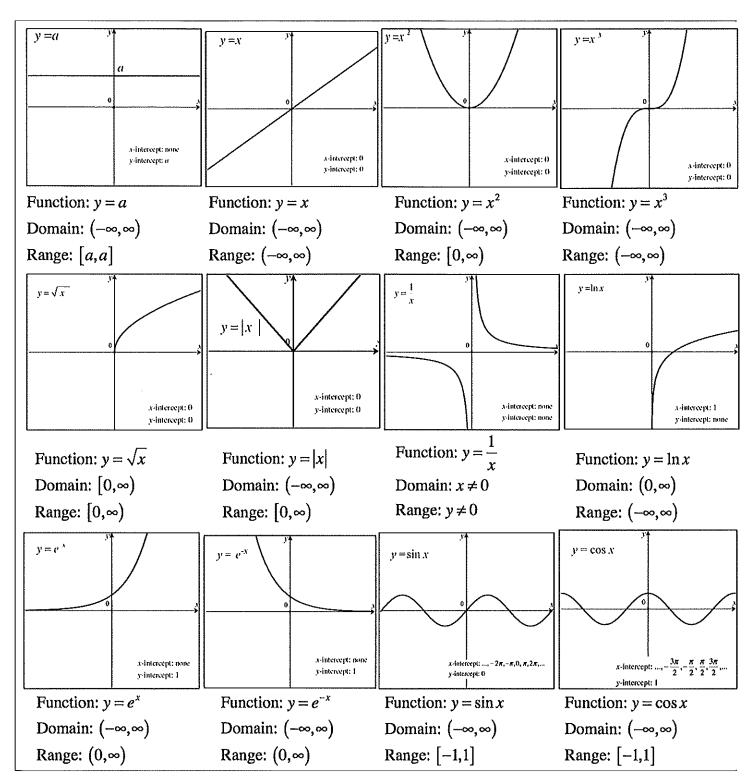


139.

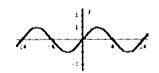


140.

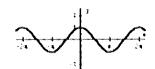
O. Graphs of Common Functions (Remember these!):



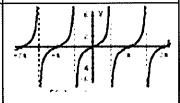
Trigonometric Functions →



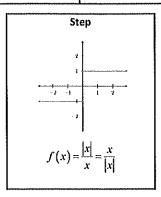
$$f(x) = \sin(x)$$



$$f(x) = \cos(x)$$



$$f(x) = tan(x)$$



P. Logarithms

If
$$y = b^x$$
, then $x = \log_b y$.

It the base of a log is not specified, it is defined to be 10. When we are asked to solve $\log 100$, we are solving the equation: $10^x = 100$ and x = 2.

$$\ln e = 1
\ln 1 = 0
e^{\ln x} = x$$

The function $y = \log x$ has domain $(0, \infty)$ and range $(-\infty, \infty)$.

Finding $\ln 5$ is the same as solving the equation $e^x = 5$.

Rules for simplifying problems involving logs and natural logs (In) are below. These rules work with logs of any base including natural logs.

$$\log a + \log b = \log(a \cdot b)$$

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

$$\log a^b = b \log a$$

$$\log_b b = 1$$

$$\log_b b^n = n$$

$$b^{\log_b n} = n$$

For #s 141 - 154, evaluate.

142.
$$\ln \sqrt{e}$$

144.
$$3^{2x} = 5^{x+1}$$

145.
$$\log 2 + \log 50$$

146.
$$\log_4 192 - \log_4 3$$

147.
$$\ln \sqrt[5]{e^3}$$

148.
$$\log_2 \frac{1}{4}$$

150.
$$\ln \frac{1}{\sqrt[3]{e^2}}$$

151.
$$e^{\ln 12}$$

152.
$$\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$$

153.
$$\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$$

154.
$$\log_3 \sqrt{3}^5$$

For #s 155 - 175, solve the equation. Check for extraneous solutions.

155.
$$5^{x+1} = 25$$

156.
$$\frac{1}{3} = 3^{2x+2}$$

157.
$$\log_2 x = 3$$

158.
$$\log_3 x^2 = 2\log_3 4 - 4\log_3 5$$

159.
$$\log_9(x^2 - x + 3) = \frac{1}{2}$$

160.
$$\log_{36} x + \log_{36} (x - 1) = \frac{1}{2}$$

161.
$$\ln x - \ln(x - 1) = 1$$

162.
$$5^x = 20$$

163.
$$e^{-2x} = 5$$

164.
$$2^x = 3^{x-1}$$

165.
$$\log(x+2) + \log x = 3\log 2$$

166.
$$3 \ln 5x = 10$$

167.
$$\log(x+4) - \log x = \log(x+2)$$

168.
$$\log_3 x + \log_3(x^2 - 8) = \log_3 8x$$

169.
$$\log_5(3x - 8) = 2$$

170.
$$\log_2(x-1) + \log_2(x+3) = 5$$

171.
$$\log_5(x+3) - \log_5 x = 2$$

172.
$$\ln x^3 - \ln x^2 = \frac{1}{2}$$

173.
$$3^{x-2} = 18$$

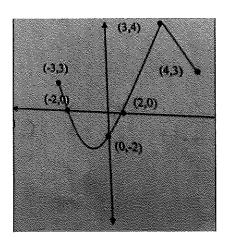
174.
$$e^{3x+1} = 10$$

175.
$$8^x = 5^{2x-1}$$

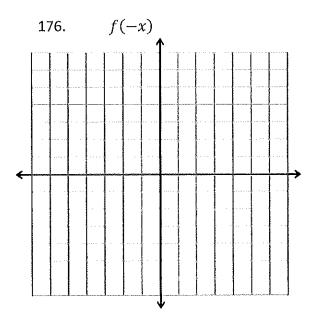
Q. Transformations/ Graphing

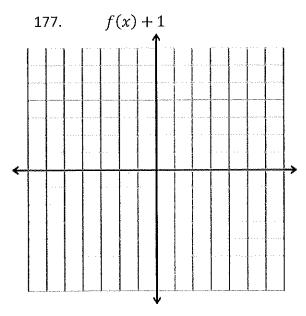
Notation	How $f(x)$ changes	Example with $f(x) = x^2$
f(x)+a	Moves graph up a units	
f(x)-a	Moves graph down a units	
f(x+a)	Moves graph a units left	
f(x-a)	Moves graph a units right	
$a \cdot f(x)$	a > 1: Vertical Stretch	
$a \cdot f(x)$	0 < a < 1: Vertical shrink	

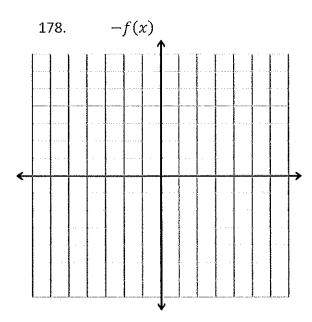
f(ax)	a >1: Horizontal compress (same effect as vertical stretch)	
f(ax)	0 < a < 1: Horizontal elongated (same effect as vertical shrink)	
-f(x)	Reflection across x-axis	
f(-x)	Reflection across y-axis	

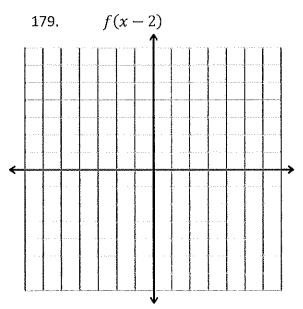


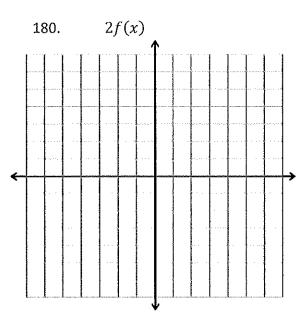
For #s 176 - 183, use the graph of f(x), given above. Sketch each transformation.

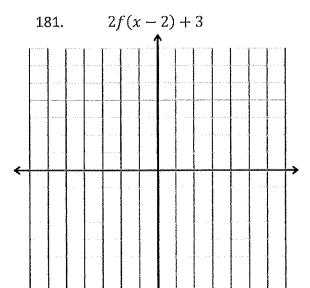


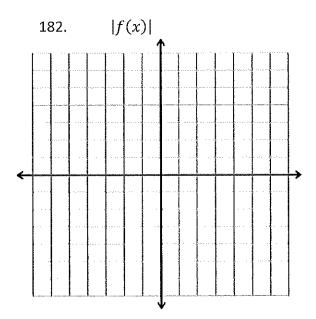


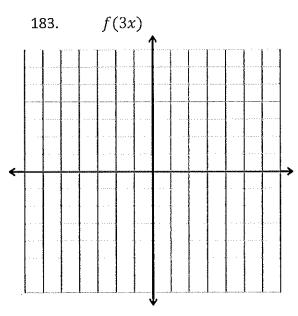












Vertical Asymptotes:

Rational functions in the form $y=\frac{p(x)}{q(x)}$ possibly have vertical asymptotes, lines that the graph of the curve approach but never cross.

To find the vertical asymptote(s) of the function, factor out any common factors of the numerator and denominator, reduce if possible (factors that cancel become HOLES), and then set the denominator equal to 0 and solve to find the x-value for which the function is undefined. That will be the vertical asymptote.

Horizontal Asymptotes:

Horizontal asymptotes are lines that the graph of the function approaches when x gets very large or very small.

To find the horizontal asymptote of the function, use one of the three cases below:

Case I: Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II: Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

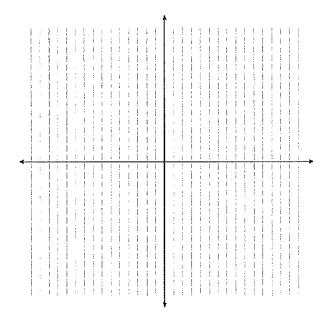
Case III: Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound.

If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.

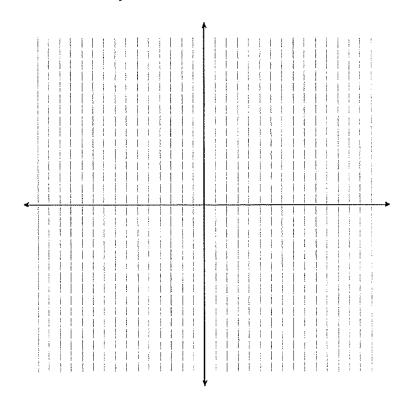
To find the limit as x goes to infinity, you use the same process as finding the horizontal asymptote.

For #s 184 - 201, sketch the equation. If applicable, find the vertical, horizontal, and slant asymptotes, holes, and x - and y - intercepts. State where the function is increasing and decreasing.

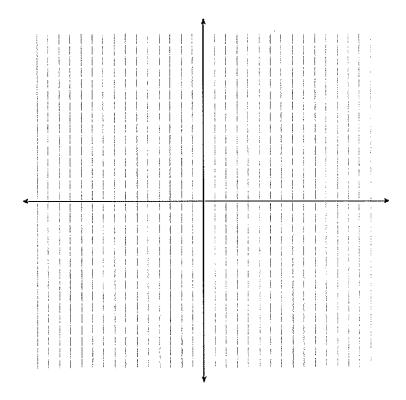
184.
$$y = -x^2$$



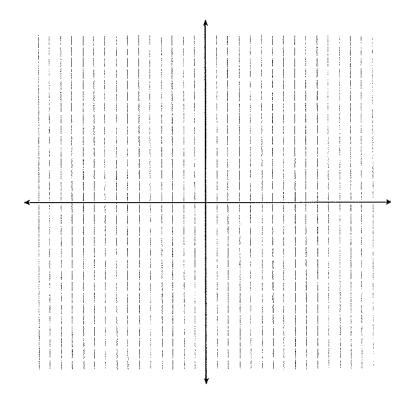
185.
$$y = 2x^2 - 4x + 1$$



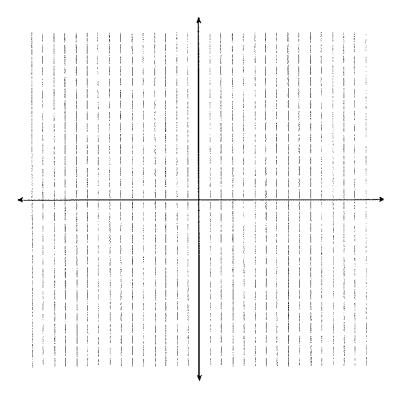
186.
$$y = -(x-2)^2 - 3$$



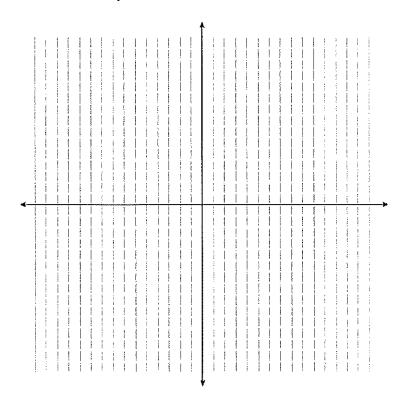
187.
$$y = 2 - \sqrt{x}$$



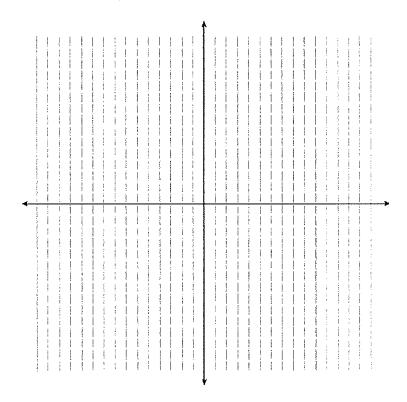
188.
$$y = \sqrt{x+1} + 1$$



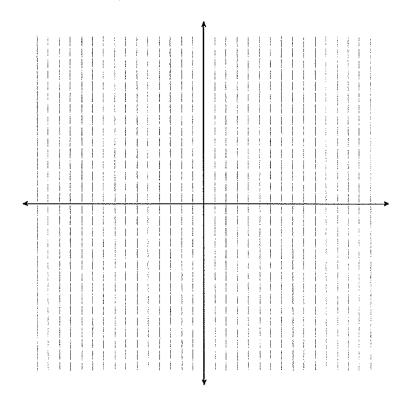
$$189. y = \sqrt{4x} + 4$$



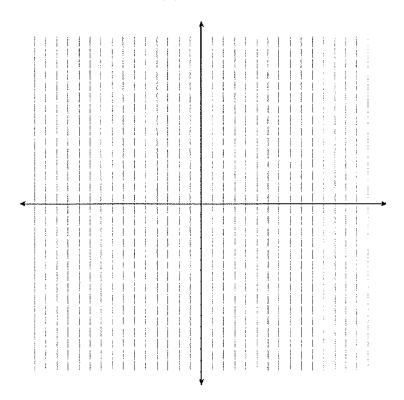
190.
$$y = |x+1| - 3$$



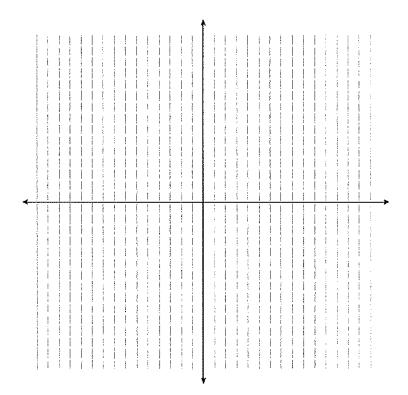
191.
$$y = -2|x-1| + 4$$



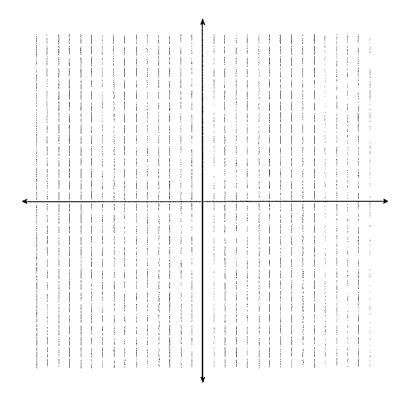
192.
$$y = -\left|\frac{x}{2}\right| - 1$$



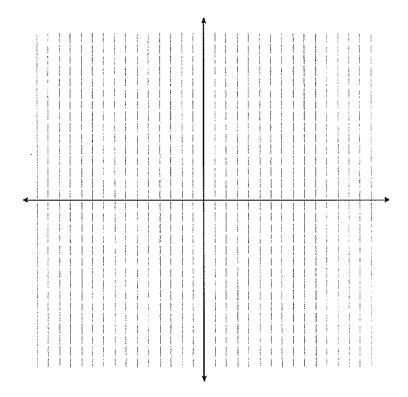
193.
$$y = \frac{-2}{x+1}$$



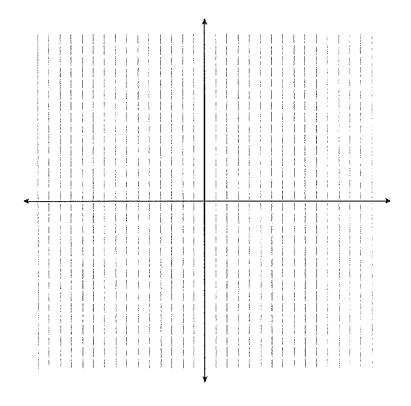
194.
$$y = \frac{1}{(x+2)^2} - 3$$



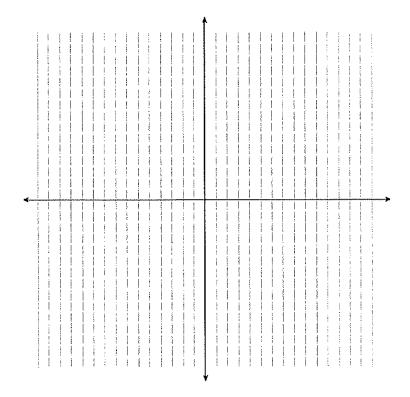
195.
$$f(x) = \frac{x^2 - 3x}{x^3 + 2x^2 - 8x}$$



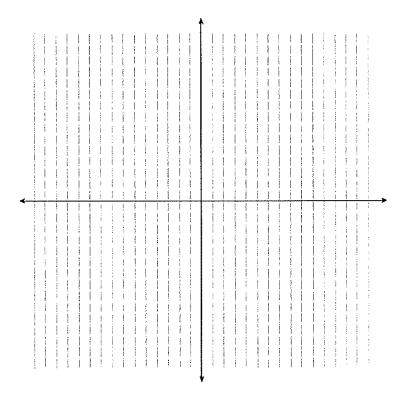
196.
$$f(x) = \frac{-3x^2 + x + 10}{3x + 5}$$

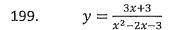


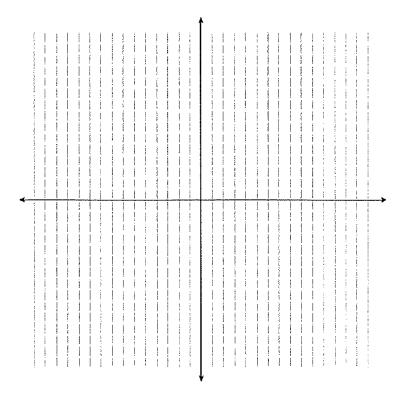
197.
$$f(x) = \frac{x^2}{x^2 - 4}$$

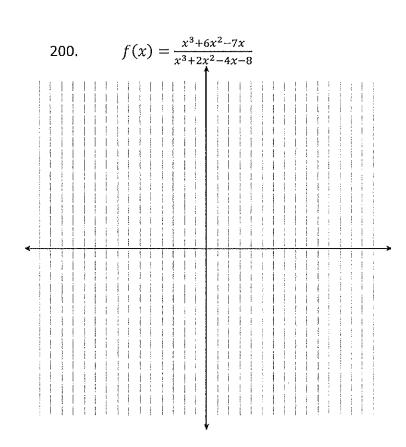


198.
$$f(x) = \frac{x^3 + 2x^2 - 15x}{x^2 - 2x - 3}$$

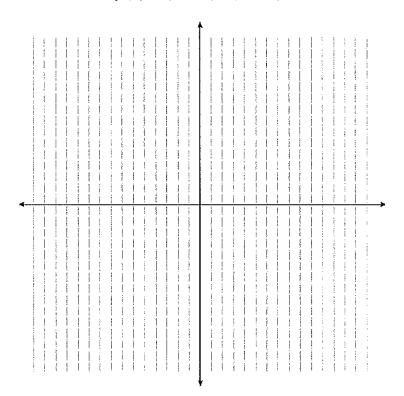








f(x) = |x+3| - |x-5|201.



R. Exponents

Negative powers do not make expressions negative; they create fractions.

$$x^{-n}=\frac{1}{x^n}, x\neq 0$$

Fractional exponents create roots.

$$x^{1/2} = \sqrt{x}$$
 and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

When we multiply, we add exponents: $(x^a)ig(x^big)=x^{a+b}$

When we divide, we subtract exponents: $\frac{x^a}{x^b} = x^{a-b}$ or $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}x \neq 0$ When we raise powers, we multiply exponents: $(x^a)^b = x^{ab}$

For #s 202 - 218, simplify the expression. Write your final answer using only positive exponents.

202.
$$-8x^{-2}$$

203.
$$(-5x^3)^{-2}$$

$$204. \qquad \left(\frac{-3}{x^4}\right)^{-2}$$

205.
$$(16x^{-2})^{\frac{3}{4}}$$

206.
$$(4x^2 - 12x + 9)^{-\frac{1}{2}}$$

207.
$$\frac{-2}{3}(8x)^{-\frac{5}{3}}(8)$$

208.
$$\frac{(x+4)^{1/2}}{(x-4)^{-1/2}}$$

209.
$$(x^{-1} + y^{-1})^{-1}$$

210.
$$3x^2(2z^3)^2$$

211.
$$(x-2)^{-4}(x-2)^{-3}$$

$$212. \qquad \left(\frac{x^{-2}y^2}{3}\right)^{-1}$$

$$213. 9x\sqrt{8x} - 3\sqrt{2x^3}$$

214.
$$-5\sqrt{16y} + 10\sqrt{y}$$

$$215. \qquad \sqrt[3]{\frac{16}{v^5}}$$

216.
$$2\left(\frac{2}{2-x}\right)\left[-\frac{2}{(2-x)^2}\right]^{-3}$$

217.
$$\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$$

218.
$$\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$$

S. Factoring

Greatest common factor: $x^3 + x^2 + x = x(x^2 + x + 1)$ Difference of squares: $x^2 - y^2 = (x + y)(x - y)$ Perfect square trinomial: $x^2 + 2xy + y^2 = (x + y)^2$ Perfect square trinomial: $x^2 - 2xy + y^2 = (x - y)^2$ Sum of two cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ Difference of two cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ Grouping: xy + xb + ay + ab = x(y + b) + a(y + b) = (x + a)(y + b)

For #s 219 - 243, factor the expression.

219.
$$4a^2 + 2a$$

220.
$$x^2 + 16x + 64$$

221.
$$4x^2 - 64$$

222.
$$5x^4 - 5y^4$$

223.
$$16x^2 - 8x + 1$$

224.
$$9a^4 - a^2b^2$$

$$20x^2 - 40x + 200$$

226.
$$x^3 - 8$$

227.
$$8x^3 + 27y^3$$

228.
$$x^4 + 11x^2 - 80$$

229.
$$x^4 - 10x^2 + 9$$

230.
$$36x^2 - 64$$

231.
$$x^3 - x^2 + 3x - 3$$

232.
$$x^3 + 5x^2 - 4x - 20$$

233.
$$9 - (x^2 + 2xy + y^2)$$

234.
$$30x - 9x^2 - 25$$

235.
$$3x^3 - 3$$

236.
$$16x^4 - 24x^2y + 9y^2$$

237.
$$x^6 - 1$$

238.
$$256x^{8z} - y^{8t}$$

239.
$$4x^2 - 12xy + 9y^2 - 49z^2$$

240.
$$x^2 - 8xy + 16y^2 - 25$$

241.
$$81x^{4b} - y^{8b}$$

242.
$$x^3 - xy^2 + x^2y - y^3$$

243.
$$(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$$

T. Completing the Square

For #s 244 - 247, write each equation in vertex form. Identify the vertex.

244.
$$y = x^2 - 4x + 8$$

$$245. x^2 + 4x = -45 + x$$

$$246. y = 3x^2 + 12x - 9$$

247.
$$y = 2x^2 - 6$$

U. Solving Quadratic Equations

For solving quadratic equations, remember:

The quadratic formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant, $b^2 - 4ac$, will tell you how many solutions the quadratic has:

$$b^2 - 4ac = \begin{cases} > 0, 2 \text{ real solutions} \\ = 0, 1 \text{ real solution} \\ < 0, 0 \text{ real solutions} \text{ (or 2 imaginary solutions)} \end{cases}$$

Solve. Factor or use the quadratic equation.

$$248. x^2 - 10x + 25 = 0$$

$$249. 12x^2 + 23x = -10$$

250.
$$8x - 3x^2 = 2$$

$$251. 6x^2 + 5x + 3 = 0$$

$$252. x^4 + x^2 - 6 = 0$$

253.
$$3(y+1)^2 - 2 = 0$$

V. Inverses

To find the inverse of a function, switch x and y and solve for the new y.

To prove that one function is an inverse of another function, you need to show that f(g(x)) = g(f(x)) = x.

Find the inverse of each function.

254.
$$f(x) = 2x + 1$$

255.
$$f(x) = \frac{x^2}{3}$$

Prove that f and g are inverses of each other.

256.
$$f(x) = \frac{x^3}{2}$$
; $g(x) = \sqrt[3]{2x}$

257.
$$f(x) = 9 - x^2, x \ge 0; g(x) = \sqrt{9 - x}$$

W. Equation of a Line

Slope of a line: $m=\frac{rise}{run}=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$, where (x_1,y_1) and (x_2,y_2) are the two points.

Slope-intercept form: y = mx + b, where m = slope and b = y-intercept.

Point-slope form: $y - y_1 = m(x - x_1)$, where m = slope and (x_1, y_1) is a point on the line.

General form: Ax + By = C, where A, B, and C are integers, and A is a positive integer.

Remember, the acronym H0Y VUX:

- H: Horizontal lines have
- 0: Zero slope and
- y =equation

V: Vertical lines have

U: Undefined slopes and

x = equation

Write the equation of the line with the given conditions.

Use slope-intercept form to find the equation of the line having a slope of 3 and a *y*-intercept of 5.

259. Find the equation of the line passing through the point (5, -3) with an undefined slope.

260. Find the equation of the line passing through the point (-4, 2) with a slope of 0.

Use slope-intercept form to find the equation of the line passing through $(\frac{3}{4}, -1)$ and $(1, \frac{1}{2})$.

- Use point-slope form to find the equation of the line passing through the point (2,5) with a slope of $\frac{2}{3}$.
- 263. Use point-slope form to find the equation of the line passing through the point (2,8) and parallel to the line $y = \frac{5}{6}x 1$.
- Use point-slope form to find the equation of the line perpendicular to the y-axis, passing through the point (4,7).
- Use point-slope form to find the equation of the line passing through the points (-3,6) and (1,2).
- Use point-slope form to find the equation of the line with an x-intercept (2,0) and a y-intercept (0,3).
- Use point-slope form to find the equation of the line containing (4, -2) and parallel to the line containing (-1, 4) and (2, 3).
- 268. Find k if the lines 3x 5y = 9 and 2x + ky = 11 are parallel.

269. Find k if the lines 3x - 5y = 9 and 2x + ky = 11 are perpendicular.

X. Radians & Degrees

To convert from radians to degrees, multiply by $\frac{180^\circ}{\pi \ radians}$.

To convert from degrees to radians, multiply by $\frac{\pi \ radians}{180^\circ}$.

Convert the following to degrees.

270.
$$\frac{5\pi}{6}$$

271.
$$\frac{4\pi}{5}$$

Convert the following to radians.

Y. Angles in Standard Position

Given a right triangle with one of the angles named θ , and the sides of the triangle relative to θ named opposite (y), adjacent (x), and hypotenuse (r), we define the six trig functions to be:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{r}{x}$$

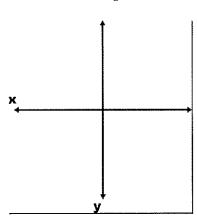
$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$$

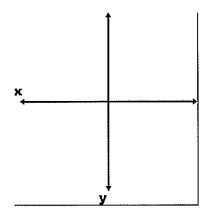
Remember: SOH CAH TOA

The Pythagorean Theorem ties these variables together: $x^2 + y^2 = r^2$

Sketch the angle in standard position.

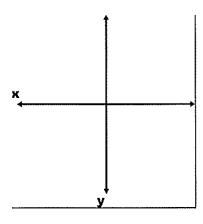






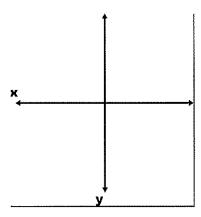
278.

 $\frac{-5\pi}{3}$



279.

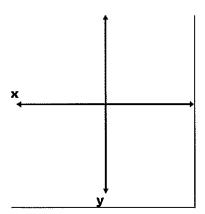
1.8 radians



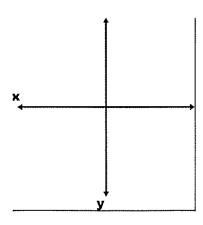
Z. Reference Triangles

Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

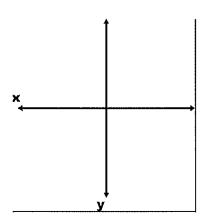
280. $\frac{2}{3}\pi$



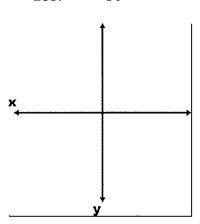
281. 225°



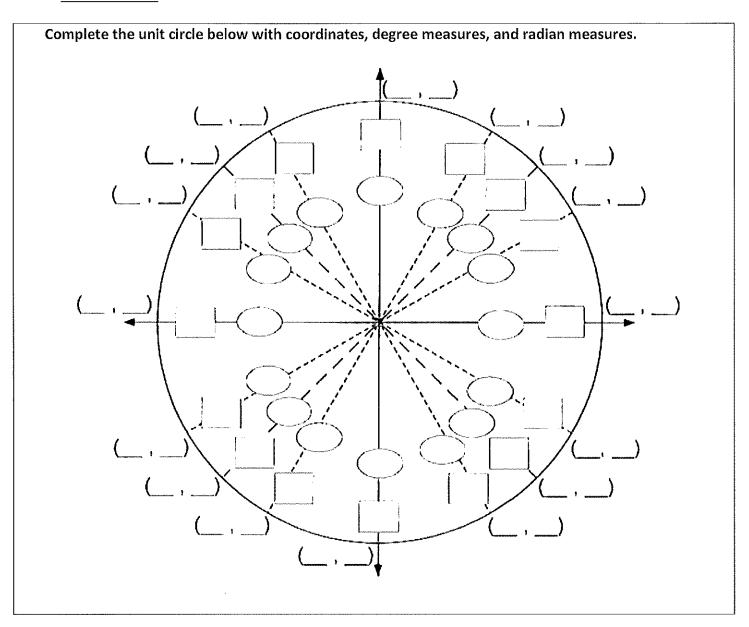
282. $-\frac{\pi}{4}$



283. 30°



AA. The Unit Circle



Find the exact value of each.

- 284. sin 210°
- 285. cos 300°
- 286. $\sin(-90^{\circ})$

287.
$$\sin \frac{2\pi}{3}$$

289.
$$\cos\left(\frac{7\pi}{6}\right)$$

290.
$$\sin\left(\frac{3\pi}{2}\right)$$

291.
$$\sec\left(-\frac{4\pi}{3}\right)$$

292.
$$\csc\left(\frac{4\pi}{3}\right)$$

293.
$$\cos\left(\frac{11\pi}{6}\right)$$

295.
$$\sin\left(-\frac{\pi}{6}\right)$$

296.
$$\csc -\frac{\pi}{3}$$

$$297. \sin\frac{\pi}{6}$$

298.
$$\tan\left(\frac{4\pi}{3}\right)$$

299.
$$\cot\left(-\frac{\pi}{4}\right)$$

300.
$$\sec\left(\frac{4\pi}{3}\right)$$

301.
$$\csc\left(\frac{3\pi}{4}\right)$$

Find the exact value of the following.

302.
$$5\cos 300^{\circ} - 4\sin 270^{\circ}$$

303.
$$\sin^2 120^\circ + \cos^2 120^\circ$$

304.
$$\cot(-30^\circ) + 3\tan 600^\circ - \csc(-450^\circ)$$

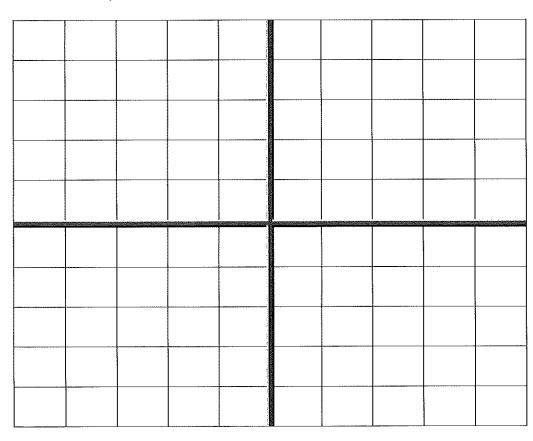
305.
$$(2\cos\pi - 5\tan\frac{7\pi}{4})^2$$

BB. Graphing Trigonometric Functions

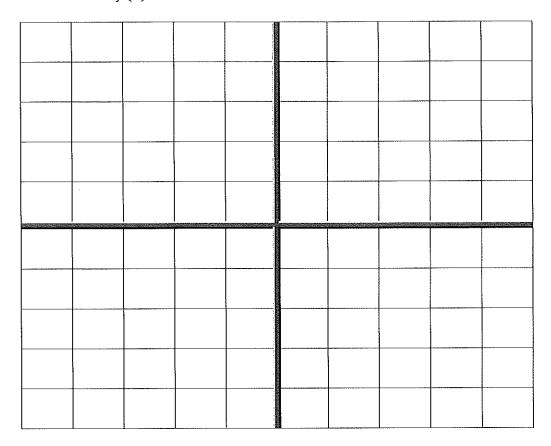
 $y=\sin x$ and $y=\cos x$ have a period of 2π and an amplitude of 1. For $f(x)=A\sin(Bx+C)+K$, A=amplitude, $\frac{2\pi}{B}=$ period, $\frac{C}{B}=$ phase shift (positive $\frac{C}{B}$ means shift left, negative $\frac{C}{B}$ means shift right) and K = vertical shift.

Complete one complete period of the function.

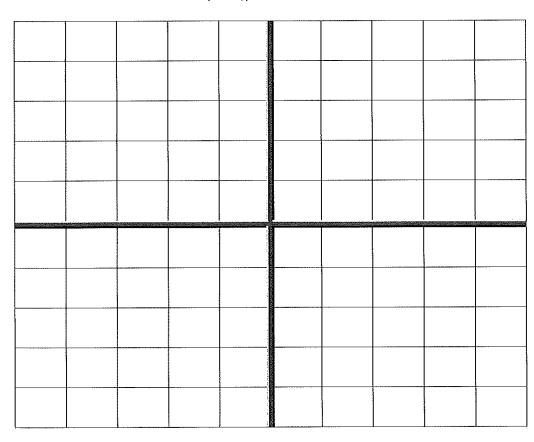
$$306. \qquad f(x) = 5\sin x$$



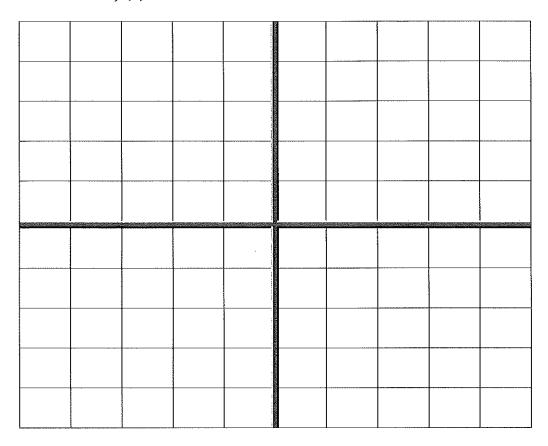
$$307. f(x) = \sin 2x$$



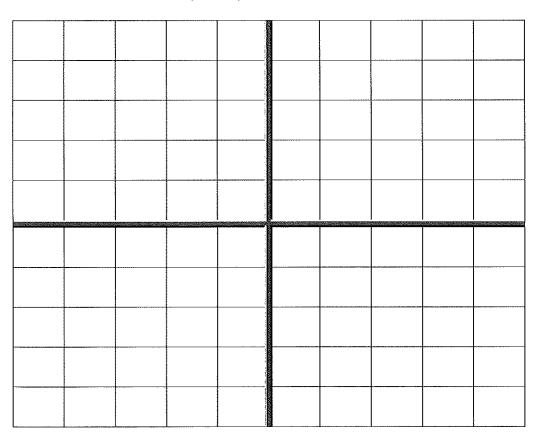
$$308. f(x) = -\cos\left(x - \frac{\pi}{4}\right)$$



$$309. \qquad f(x) = \cos x - 3$$



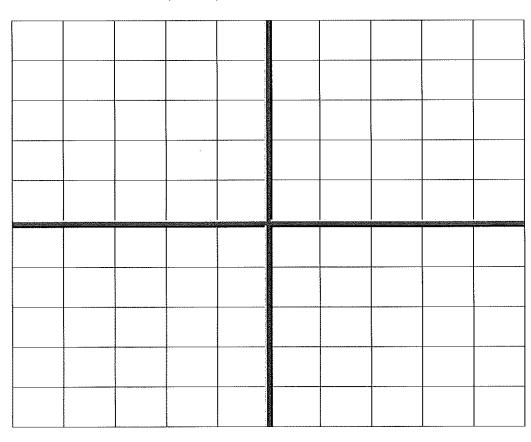
$$310. y = -3\sin\left(4x - \frac{\pi}{2}\right)$$



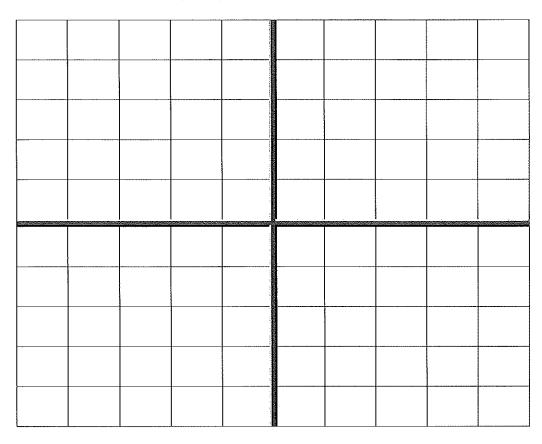
311.
$$y = 4\cos(3x + \pi)$$

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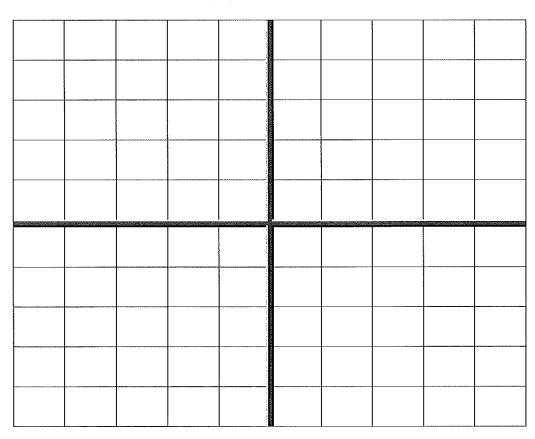
312.
$$y = \tan\left(2x - \frac{\pi}{2}\right)$$



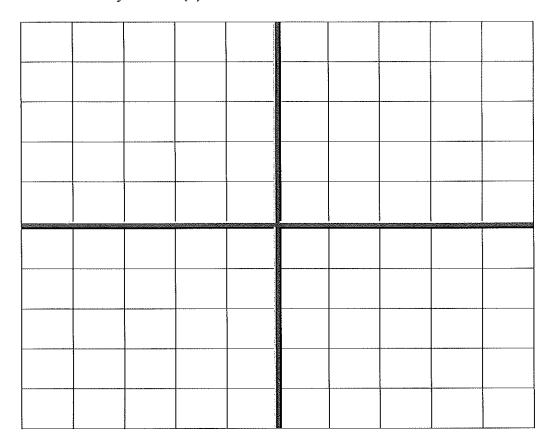
313.
$$y = \frac{1}{2}\csc\left(x + \frac{\pi}{4}\right)$$



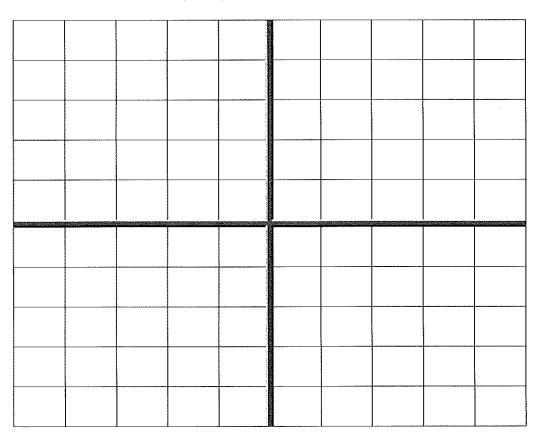
314.
$$y = 1 + 2\sec\left(\frac{1}{2}x\right)$$



315.
$$y = 2 \tan(x) - 1$$



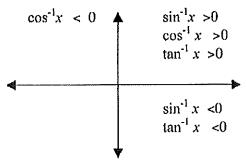
$$316. y = -\csc\left(x + \frac{\pi}{3}\right)$$



CC. Inverse Trigonometric Functions

Inverse trigonometric functions can be written in one of two ways: arcsin(x) or $sin^{-1}(x)$.

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Find the exact value of each expression.

317.
$$\tan(\arccos\frac{2}{3})$$

318.
$$\sec\left(\sin^{-1}\frac{12}{13}\right)$$

319.
$$\sin(\arctan\frac{12}{5})$$

320.
$$\sin(\sin^{-1}\frac{7}{8})$$

For each of the following, express the value of "y" in radians.

321.
$$y = \arcsin \frac{-\sqrt{3}}{2}$$

322.
$$y = \arccos(-1)$$

323.
$$y = \arctan(-1)$$

DD. Right Angle Trigonometry

Let P be a point on the terminal side of θ . Find the six trig functions of θ .

324.
$$P(-8,6)$$

325.
$$P(1,3)$$

326.
$$P(-\sqrt{10}, -\sqrt{6})$$

327. If
$$\cos \frac{2}{3}$$
, θ in quadrant *IV*, find $\sin \theta$ and $\tan \theta$.

328. If
$$\sec \theta = \sqrt{3}$$
, find $\sin \theta$ and $\tan \theta$.

EE. Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$.

$$329. \tan^2 x + \tan x = 0$$

330.
$$4\cos^2 x - 3 = 0$$

$$331. \qquad \sin(2x) - \cos x = 0$$

332.
$$\csc^2 x - \csc x - 2 = 0$$

$$333. \qquad \sin x - \cos x = 0$$

334.
$$2\sin^3 x - \sin x = 0$$

335.
$$3 \cot^2 x - 1 = 0$$

336.
$$\tan^2 x + \sec x - 1 = 0$$

337.
$$\cos x + \sin x \tan x = 2$$

FF. Circles

338. Write an equation for the circle with center at (1,2) that passes through the point (-2,-1).

339. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find the center and the radius.

340. Find the equation of a circle centered at (5, -1) with radius 4.

341. Find the center and radius of the circle: $x^2 + y^2 + 4x - 6y = -4$

- GG. Polynomial Long Division/ Synthetic Division
- 342. Determine the remainder when $x^3 6x^2 + 5x 2$ is divided by x 6.

343. Determine all the possible rational roots of $f(x) = 2x^3 - 5x^2 + 4x + 6$.

344. Is
$$(x-2)$$
 a factor of $x^3 - x^2 - x - 2$?

345. Using long division, find
$$(3x^4 - 2 - 5x) \div (x^2 - 3x)$$
.

346. Using long division, find
$$(4x^3 - x - 9) \div (2x - 3)$$
.

347. Using synthetic division, find
$$(2x^4 - 5x^2 + 6x - 9) \div (x + 2)$$
.

HH. Sum & Difference Formulas, Double-Angle Formulas, & Half-Angle Formulas

Sum & Difference Formulas

 $\sin(u+v) = \sin u \cos v + \cos u \sin v$

 $\sin(u - v) = \sin u \cos v - \cos u \sin v$

 $\cos(u+v) = \cos u \cos v - \sin u \sin v$

 $\cos(u - v) = \cos u \cos v + \sin u \sin v$

 $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

 $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

Double-Angle Formulas

 $\sin 2u = 2 \sin u \cos u$

 $\cos 2u = \cos^2 u - \sin^2 u$ $=2\cos^2 u-1$ $= 1 - 2\sin^2 u$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Half-Angle Formulas

 $\sin\frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$

 $\cos\frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$

 $\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u}$ $\tan \frac{u}{2} = \frac{1 + \cos u}{1 + \cos u}$ The signs of $\sin \frac{u}{2}$ and

 $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Find the exact value of the expression.

tan 15° 348.

349. cos 67.5°

 $\cos \frac{7\pi}{8}$ 350.

 $\sin\left(-\frac{7\pi}{12}\right)$ 351.

Find the exact vales of $\sin 2x$, $\cos 2x$, and $\tan 2x$.

352.
$$\tan x = -\frac{1}{2}, -\frac{\pi}{2} < x < 0$$

353.
$$\sin x = -\frac{3}{5}, \frac{3\pi}{2} < x < 2\pi$$

354.
$$\cos x = -\frac{1}{3}, \pi < x < \frac{3\pi}{2}$$

II. Calculator Questions (Calculator Allowed)

Using a calculator:

355. Find the relative maxima and minima of $f(x) = 2x^3 - 7x^2 - 4x$.

356. Find the relative maxima and minima of $h(x) = 2x^5 - 3x^4 + x - 4$.

357. Find the zeros of $3x^3 - x - 5 = 0$.

358. Find the zeros of $2x^2 - 1 = 2^x$.

359. Find the zeros of $2 \ln(x + 1) = 5 \cos x$ on $[0, 2\pi)$.

360. Find the intersection of $\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2\\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$.

361. Find the distance between -5.4 and $3\frac{3}{4}$.

362. Find the distance between (-3,5) and (-6,-2).

Determine an equation of a polynomial with degree 3 and zeros of -3, 2, and $\frac{3}{2}$.

364. If $y = x^2 + kx - k$, for what values of k will the quadratic have two real solutions?

365. The function $C(x) = \frac{10}{2x^2 + 1}$ can be used to find the concentration C(x) in mg/L of a certain drug in the bloodstream of a patient x hours after the injection is given. In approximately how many hours after the injection will the concentration of the drug be approximately 1.3 mg/L?

Use your calculator to complete the first problem in the space provided. Circle your answer. Find your answer among the choices. Put #2 in the problem blank. Work that question and proceed in this manner until finished. You may use any of the tools on your calculator to solve these problems.

Answer: 4.272	Answer: 1.024
#1 Evaluate: ³ √76.5	# Solve for x. $ x^3 - 4x = 7 - x$
	To advance in the circuit, find the sum of the two solutions.
	two solutions.
Answer: 0.813	Answer: 4.277
# Find the minimum value of the	# Let $f(x) = e^{x-4} + 2.5x - 11.7$. Find
function $h(x) = 1 + x + e^{x^2 + 3x}$.	the zero of the function.
function n(x) = 1 + x + e	the zero of the function.
Answer: 1.527	Answer: -0.144
# Solve for x on the closed interval	# Solve for x. $(2x+1)^{-2} = 10 - e^{x^2+2}$
$[2,4]. \frac{20}{3 + e^{\tan x}} = 5.3$	Solve for x. (2x + 1) = 10 - c
	There are two solutions. To advance in the circuit, find the smallest solution.
Answer: 6.990	Answer: 1.682
# If $f(x) = \ln(x+4)$ and	# Evaluate: ln(5.86)
	#Evaluate, III(5.50)
$g(x) = \tan(x^2)$, find $f(g(3.2))$.	
Answer: 0.456	Answer: -1.256
	# If $f(x) = x^5 - 2x^4 + \sin^2 x + k$,
# If $h(x) = \begin{cases} x \sec x, & x \le 1 \\ x \tan^{-1} x, & x > 1 \end{cases}$ find	
•	find k so that $f(2.1) = 1.212$.
h(0.9) and $h(1.1)$.	
To advance in the circuit, find the largest of the two values.	

Answer: 1.622	Answer: 1.768
# Solve for x . $\frac{2}{x+2} - \frac{7}{x-5} = 10$	# If $f(x) = 4.5x^3 - 3.2x^2 - \sin x$, find $f(1.5)$.
There are two solutions. To advance in the circuit, find the positive solution. Answer: -0.321 # Solve for x . $ 3x-4 =2.5\sqrt{3-x}$	Answer: -1.478 # If the radius of a cone is 0.9 inches and the height is twice the radius, what is the volume (in inches³) of the cone? $\left(V = \frac{1}{3}\pi r^2 h\right)$
There are two solutions. To advance in the circuit, find the solution closest to zero.	1
Answer: 4.245 # Evaluate: (51.4) ^{3/7}	Answer: 1.448 # If the volume of a sphere is 4.5 m³, find the radius of the sphere. $\left(V = \frac{4}{3}\pi r^3\right)$
Answer: 2.890 # A remote control plane climbs at takeoff with a slope $m = 0.178$. How far off the ground is the plane when it has traveled 24 feet in the horizontal direction after takeoff?	Answer: -0.176 # Find the maximum value of the function $g(x) = \frac{4.3x}{x^2 + 7}$
Answer: 4.194 # If $g(x) = \sin^2(2x)$, find $g(1.2)$.	Answer: 5.411 # Evaluate: e ^{0.52}